UDC 621.373.1
DOI https://doi.org/10.32782/2521-6643-2025-1-69.32

Semenov A. O., Doctor of Technical Sciences, Professor Department of Information Radioelectronic Technologies and Systems
Vinnytsia National Technical University
ORCID: 0000-0001-9580-6602

Khloba A. A., Postgraduate student Department of Information Radioelectronic Technologies and Systems
Vinnytsia National Technical University
ORCID: 0009-0007-6743-2456

CONTROL DEVICES FOR PROJECTION REFLECTIONS IN THE PHASE SPACE OF STRANGE ATTRACTORS OF DYNAMIC CHAOS GENERATORS

One of the ways to experimentally test the proposed mathematical models that exhibit the properties of deterministic chaos is to implement them using «analog computers». Depending on the dimensionality of the system, three or more signals are obtained, the spectrum of which can be analyzed. However, the most obvious evidence of chaotic behavior is strange attractors. A two-dimensional version of attractors is obtained by applying two signals to an oscilloscope that is switched to the X-Y mode. In this way, three projections are obtained, although the minimum dimensionality of the system implies that the object is three-dimensional. There are digital ways of displaying strange attractors in 3D using special consoles; they are connected to the system under study and to a personal computer. The greater the accuracy of such a set-top box, the higher its cost. However, it is possible to implement the display of a strange attractor on the oscilloscope screen in pseudo-3D using simple mathematical operations. With this approach, no information is lost during signal processing, and the cost of the device is lower. The structure of the object of study can be compared to a mathematical simulation by rotating it in phase space immediately after connecting three signals to the console, without additional programs. The basic mathematical operations are realized with the help of operational amplifiers, inverters, analog multipliers, and sin/cos potentiometer analogs. The article is devoted to a number of devices-attachments to the oscilloscope that make it possible to rotate strange attractors in pseudo 3D along two or three axes. The presented works contain device schematics and the necessary information for independent implementation. These studies demonstrate the sequence of development of the idea, the gradual departure from the analog sin/cos potentiometer to its digital counterparts, and the expansion of the rotation range from 90 to 360 degrees. The possibility of drawing a sectional plane along one of the axes and obtaining a Poincaré section is controlled. The main structural elements of the devices are defined, and the operation of some of them is briefly described. For a better understanding of the operation of such devices, images illustrating rotations in phase space are shown. A certain number of images were converted from black and white to color and further processed. The prospective development of such devices has been determined.

Key words: *dynamic chaos, attractor, generator, Poincaré section, phase plane, device, signal, mathematical model.*

Семенов А. О., Хльоба А. А. Пристрої керування відображеннями проекцій у фазовому просторі дивних атракторів генераторів динамічного хаосу

Одним з способів експериментальної перевірки запропонованих математичних моделей, які проявляють властивості динамічного хаосу, є реалізація їх за допомогою «аналогових комп'ютерів». В залежності від розмірності системи отримують три та більше сигналів, спектр яких можна проаналізувати. Але найбільш наглядним доказом хаотичної поведінки є дивні атрактори. Двовимірну версію атракторів отримують шляхом подачі двох сигналів на осцилограф, який переведений в режим X-Y. Таким чином отримують три проекції, хоча мінімальна розмірність системи передбачає, що об'єкт тривимірний. Існують цифрові способи відображення дивних атракторів в 3D за допомогою спеціальних приставок, вони підключаються до досліджуваної системи і до персонального комп'ютера. Чим більша точність такої приставки тим більша її вартість. Однак наявна можливість за допомогою простих математичних операцій реалізувати відображення дивного атрактору на екрані осцилографа у псевдо 3D. При такому підході не втрачається інформація при обробці сигналу, вартість приладу менша. Структуру об'єкта дослідження можна порівняти з математичною симуляцією обертаючи його в фазовому просторі відразу після підключення трьох сигналів до приставки, без додаткових програм. Основні математичні операції реалізуються за допомогою операційних підсилювачів, інверторів, аналогових помножувачів і аналогів sin/cos потенціометра. Стаття присвячена ряду пристроїв-приставок до осцилографу які дають можливість обертати дивні атрактори в псевдо 3D по двох, або трьом осям. У наведених роботах наявні схеми приладів і необхідна інформація для самостійної реалізації. Приведені дослідження *демонструють послідовність розвитку ідеї, поступовий відхід від аналогового sin/cos потенціометра до його цифрових аналогів, розширення діапазону обертання від 90 до 360 градусів. Керована можливість проведення площині перерізу по одній з осей і отримання перетину Пуанкаре. Визначено основні структурні елементи приладів, коротко описано роботу деяких з них. Для кращого розуміння роботи таких пристроїв наведені зображення які ілюструють повороти у фазовому просторі. Певна кількість зображень була перетворена з чорно білих на кольорові і додатково опрацьовано. З'ясовано перспективний розвиток таких приладів.*

Ключові слова: *динамічний хаос, атрактор, генератор, перетин Пуанкаре, фазова площина, пристрій, сигнал, математична модель.*

Formulation of the problem. The study and search for new mathematical models exhibiting the properties of dynamic chaos are often carried out not only with the help of specialized software but also by creating real prototypes of the proposed models. They can be carried out by a set of discrete elements, analog computer (operational amplifiers, analog multipliers, logic elements – carrying out mathematical operations) or with the help of programmable gate arrays (FPGA). Often the papers compare the results of: numerical modeling; simulation (PSpice, MultiSIM, OrCAD); and the characteristics of a real prototype. In case of exclusively software implementation and simplification of the mathematical model, strong discrepancies with the more real case are possible [1], a large number of differences in the software and real implementation can be found in [2] when considering strange attractors. The structure of such objects is given in 3D exclusively in the software realization, and when modulating in the simulator or on the oscilloscope screen exclusively in the X-Y mode, less often in the X-Y-Z mode. The researcher is limited to three options for displaying the strange attractor (X-Y, Y-Z, X-Z), choosing two signals from the electronic realization of the system of equations under study. This, in the case of a simple attractor shape, is sufficient for perception and comparison with the software version. However, this approach does not give an idea of depth and causes difficulties in studying the structure in the case of attractors of complex shapes [3-5].

Analysis of recent research and publications. The development of semiconductors has made operational amplifiers, various logic elements, as well as microcontrollers, FPGAs, and the development of programmable analog arrays (FPAA) [6] very accessible. The realization of dynamic chaos generators solely on discrete elements is becoming less common. Often the developed systems are tested in practice using an analog computer [2, 6]. The approach to the synthesis of such systems is reviewed in [7], the synthesis of individual elements to create nonlinear circuits is described in detail. A modular construction system as a tool for teaching problems is tested in [8], the combination of modules allows generating more than 30 strange attractors. Another paper proposes a simple and flexible analog computer for the study of linear and nonlinear systems [9]. The design of electrical circuits based on differential equations, synchronization and the use of FPAA is devoted to [10]. A hybrid analog computer capable of modeling systems up to fourth order is proposed in [11], all necessary information for its replication is available. A detailed guide to the application and use of such systems is described in [12], historically the development is given in [13]. Modern commercial versions of analog computers are proposed in [14, 15]. The implementation of the circuit using FPGA is discussed in [16]. The paper describes a way to rotate the attractor in 2D as part of an implementation on an FPGA, it is assumed that the parameter φ can be used as an additional parameter for information encoding. A development of the work is [17], now the rotation is possible in 3D using the developed algorithm CORDIC (Coordinate Rotation Digital Computer), which performs calculations of sine and cosine functions, for systems with fractional order. Realization of both works is carried out on FPGA and part of information is lost due to transformations on DAC and ADC. Also used expensive FPGA – Artix-7 XC7A100T, so this method is not universal for the study of the structure of attractors of complex shapes [18, 19].

The above-mentioned robots allow to realize the system of equations in the form of an electric circuit and display a strange attractor on the oscilloscope screen in X-Y mode. However, in none of the reviewed works, as well as numerous works devoted to the study of dynamic chaos generators, no application or description was found, as separate and universal, of devices that allow to consider strange attractors in pseudo 3D and with the ability to rotate them in phase space.

The purpose of the article. Consideration of existing methods of visualization of 3-dimensional signals using oscilloscope in X-Y mode, i.e. conversion of signals of 3D objects into pseudo 3D by displaying them on 2D oscilloscope screen. Providing some found works devoted to such devices, variants of their practical realization and use. Identification of key structural elements of such devices and prospects for further development.

Presenting main material. Rotation matrices are used to rotate a three-dimensional image by an angle φ in the Cartesian coordinate system. By multiplying the original point coordinates with one of the orthogonal matrices, new point coordinates are obtained. Rotation matrices and coordinates of points in rotation along one of the axes can be written as specified in (1). In the case of sequential rotation along two axes, the coordinates obtained at the first step are multiplied with another rotation matrix.

$$M_x(\varphi)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\varphi & -\sin\varphi\\ 0 & \sin\varphi & \cos\varphi\end{pmatrix};M_x(\varphi)\cdot\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{pmatrix}x'=x\\ y'=y\cdot\cos(\varphi)+z\cdot\sin(\varphi)\\ z'=-y\cdot\sin(\varphi)+z\cdot\cos(\varphi)\end{pmatrix}$$

$$M_y(\varphi)=\begin{pmatrix}\cos\varphi & 0 & \sin\varphi\\ 0 & 1 & 0\\ -\sin\varphi & 0 & \cos\varphi\end{pmatrix};M_y(\varphi)\cdot\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{pmatrix}x'=x\cdot\cos(\varphi)+z\cdot\sin(\varphi)\\ y'=y\\ z'=-x\cdot\sin(\varphi)+z\cdot\cos(\varphi)\end{pmatrix}\quad(1)$$

$$M_z(\varphi)=\begin{pmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{pmatrix};M_z(\varphi)\cdot\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{pmatrix}x'=x\cdot\cos(\varphi)-y\cdot\sin(\varphi)\\ y'=-x\cdot\sin(\varphi)+y\cdot\cos(\varphi)\\ z'=z\end{pmatrix}$$

where $M_n(\varphi)$ – rotation matrix along one of the axes; x,y,z – coordinates of the initial point; x′,y′,z′ – coordinates of the point rotated by some angle φ.

An example of such an approach is described in [20, p. 127]. There are several ways to perform the above transformations, in the considered work are used already practically unavailable sin/cos potentiometer (another possible option is a resolver), a simplified scheme is shown in Figure 1. To perform the operations and obtain the required sign, an inverter (to obtain -zsin(φ)) and two adders are used. Points A, B, C, D are the sin/cos leads of the potentiometer from which are taken: zcos(φ), xcos(φ), xsin(φ), zsin(φ). The case mentioned in the paper illustrates two consecutive rotations, along the Y-axis, then X′. The third obtained signal Z″ can be fed to the Z input of an oscilloscope (if available), thereby adjusting the luminance of the outlying areas. The rotation angle of the above circuit is limited to 90 degrees due to a design feature of the potentiometer.

A later work [21] also uses a sin/cos potentiometer, but to rotate the 2D image, but the input signal is not fed directly to the potentiometer, but to 4 analog multipliers, the sin/cos signal is taken from the potentiometer (or it is possible to feed signals from an external oscillator bypassing the element). Improvements to this circuit are presented in [22], it consists in reducing the noise affecting the stability of the output signal. For this purpose, the authors replace the sin/cos potentiometer with a "resistive ladder" and 4 computer-controlled multiplexers, inverters and 4 buffers.

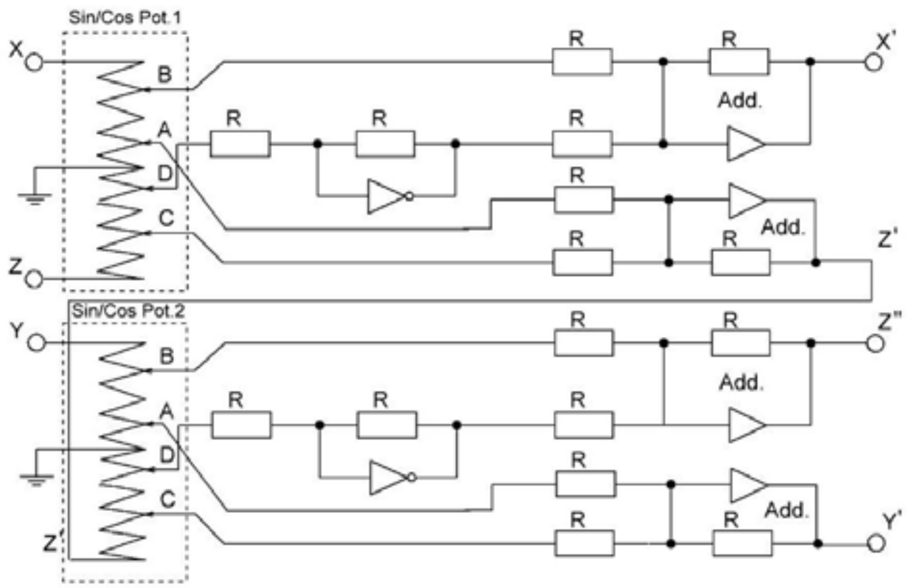


Fig. 1 Schematic diagram of the image rotation attachment based on sin/cos potentiometer in series on two axes

Thus, the circuit allows to rotate the 2D image with a step of 15 degrees (7 gradations within the first quadrant). After buffering, the summation of signals takes place at the differential inputs of the display, or, in the absence of such inputs, in the case of an oscilloscope, adders are added before the buffers. Rotation by 360 degrees is possible by supplying the opposite end of the "resistive ladder" with an anti-phase signal (-X, -Y). A different scheme is proposed in [23], the sin/cos potentiometer is replaced by a pair of EPROMs and a set of multiplying DACs to which the X, Y lines are also connected. The adjustment is done with a conventional potentiometer and ADC (controls the

EPROM pair), the rotation step is about 0.35 degrees (256 bits provide 90 degrees rotation). Two polarities of the signal (-sin(x), -cos(x)) are provided by inverters, then through multiplexers the received signal is mixed and fed to the buffer from which X′, Y′ are taken. The considered works cannot be directly used for rotation of strange attractors in 2D since the proposed schemes are designed for specific specific purposes (study of visual stimuli; rotation of an image obtained from an electron microscope). However, they demonstrate a tendency to replace sin/cos potentiometer with its digital analogs and basic blocks for performing mathematical operations using an analog computer.

Mention of further development can be found in [24], a number of methods for the study of nonlinear dynamics are given, including the work of [25]. It describes a multifunctional analog device that allows: to rotate strange attractors in pseudo-3D by 360 degrees relative to any axis continuously or manually; to perform a plane section in phase space along one of the axes; to display the upper or lower part of the split attractor; to display a Poincaré section along one of the axes by controlling the position of the section surface, and it is possible to display the "first return" in the forward and backward directions. The grid of dots indicates the reference plane (X-Y plane is accepted) for better understanding of the relative position of the projection plane. The limitation of the device is the frequency range from 0 to 20 kHz, but by replacing the microcircuits with higher-frequency ones it is possible to extend the specified range. For a more visual explanation are given the figures from this work (Figure 2, 3), they were converted from black and white to color with the help of visual-paradigm service.

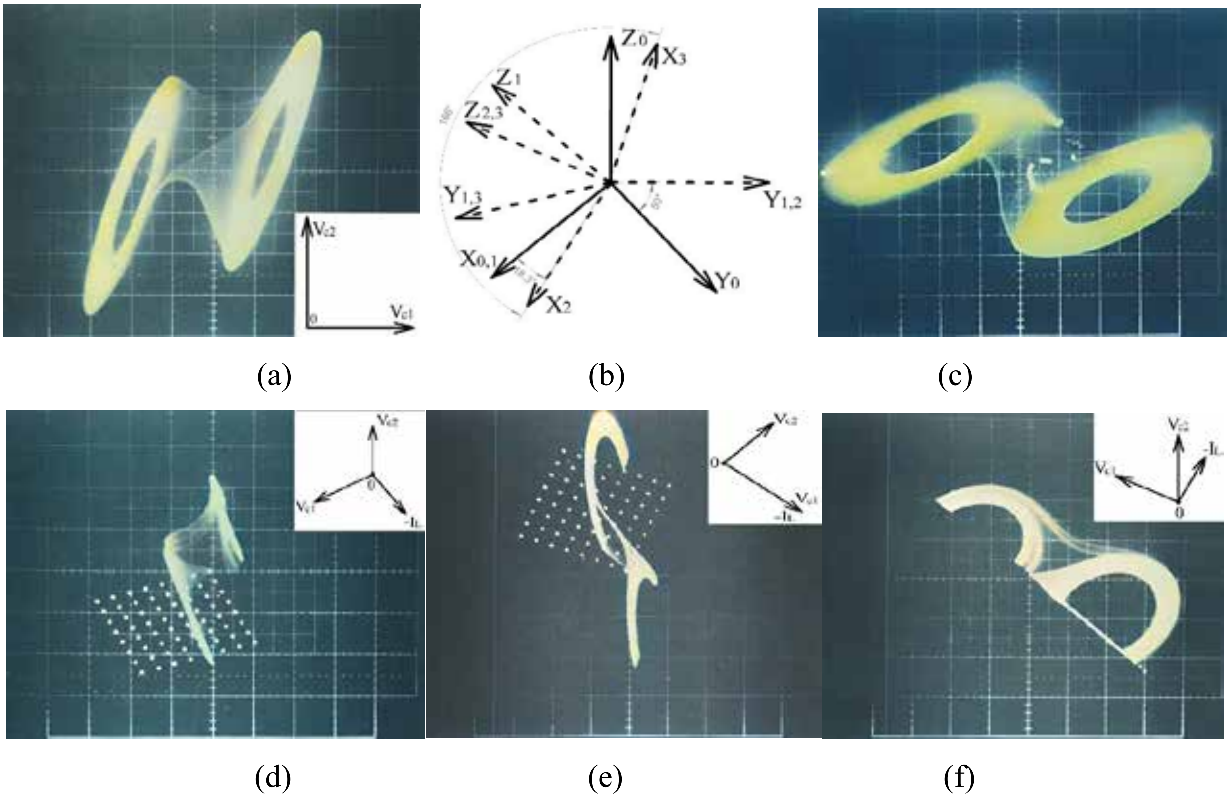


Fig. 2. Projections on the plane V_{c1}-V_{c2} (a), three-axis rotation (b), rotation by some degree, projection to X_0-Y_0 (c), top part of the attractor (a) when rotated by angles (b) at different projections (d-f) [25]

The planes on which the attractor projections fall have been added to each figure. In the case of (d-f) the position of the reference plane is indicated. The figures show a strange attractor, the signals for which are obtained from the Chua scheme [26]. The Figure 2 show the following: (a) – projection of the strange attractor on the plane V_{c1}-V_{c2} without rotation; (b) – explanation of rotation in two-dimensional space, where numbers in axis indices indicate the order of rotation with respect to axes by angles $-\varphi_x(50.6)$, $-\varphi_y(18.3)$, $\varphi_z(166)$; (c) – rotated attractor, projection on the plane X_0-Y_0, rotation angles are not specified in the work; (d-f) – upper part of the rotated attractor by angles (b) and its projections on the planes X_0-Y_0, X_0-Z_0 и Z_o-Y_o.

Figure 3 show the following: (a) – Projection of the attractor on the plane $(-I_L)$-V_{c1}, three planes of cross sections S1-S3 in phase space are shown in red color; (b-d) Poincaré cross section in forward and backward directions, trajectory directions are indicated by the red arrow (the instrument can display either or both); (e-f) – The attractor is rotated to $\varphi_z(56.3)$, is the map of the first return in the forward direction, projected on the plane X_0-Y_0. The extended structural diagram of the device is shown in Figure 4.

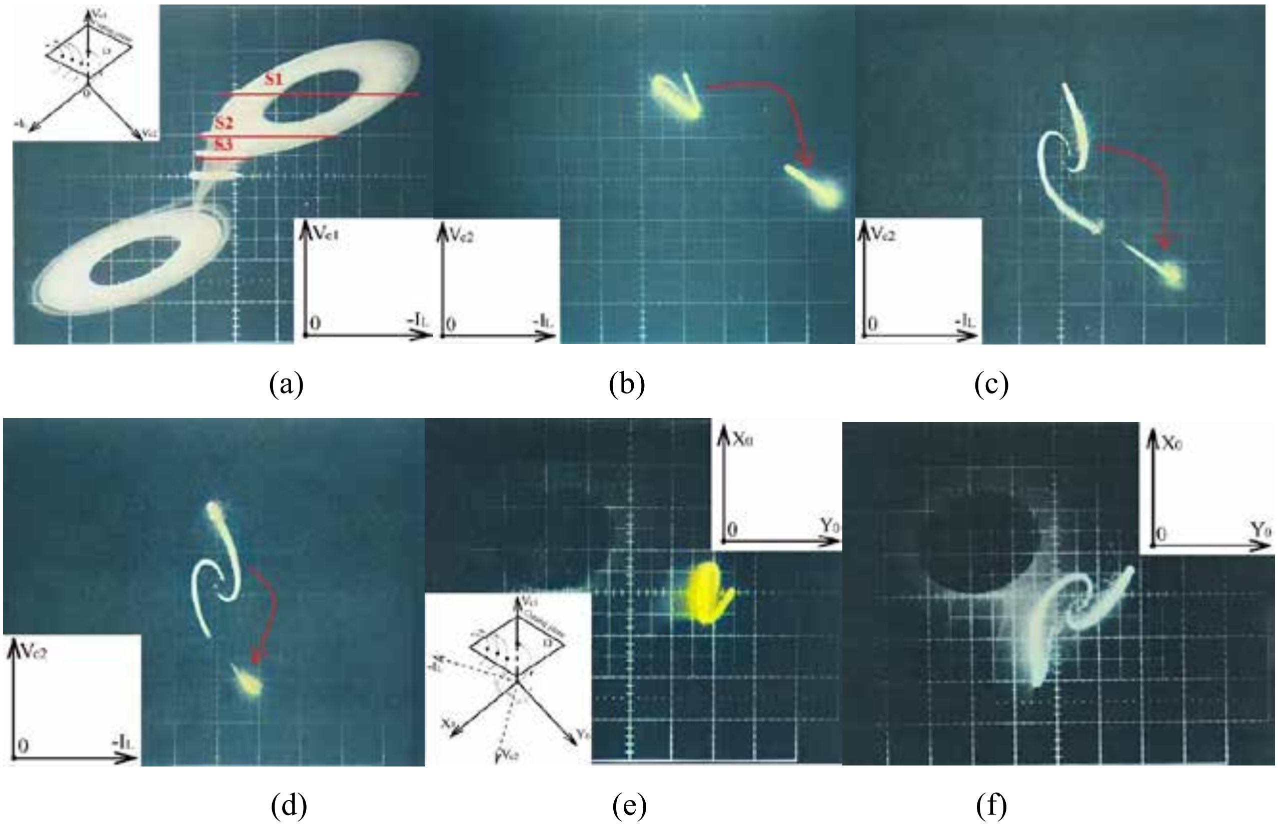


Fig. 3. Projection of an unrotated attractor in the plane $(-I_L)$-V_{c1} (a), cross sections in the phase plane S1-S3 (b-d), rotated attractor (a) and cross-sections in the phase plane S1-S2 (e-f) [25]

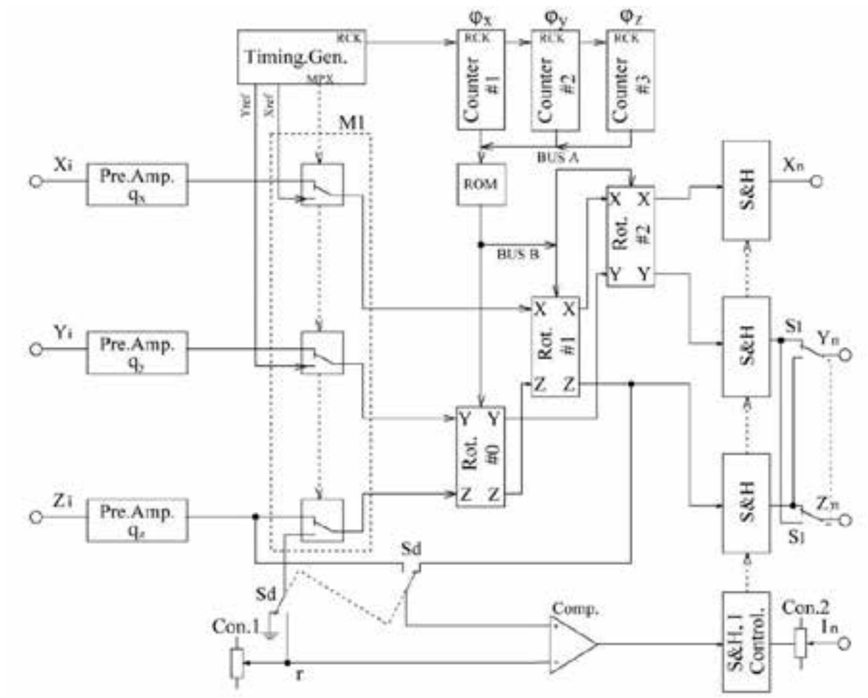
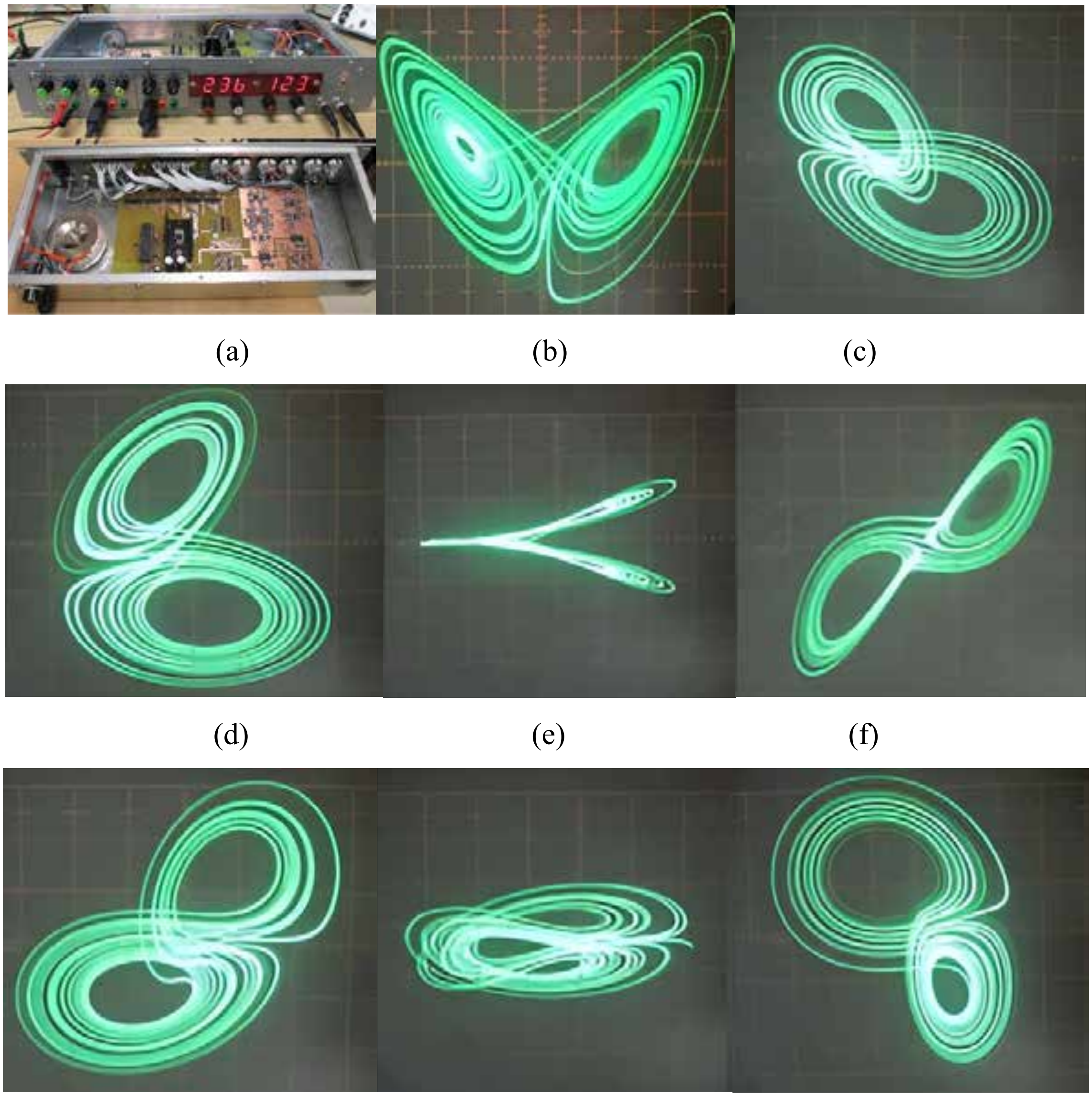


Fig. 4. Structural diagram of the device for rotation and section creation [25]

The (Pre.Amp.) blocks provide pre-scaling of the input signal. Block (Timing.Gen.) and multiplexer (M1) provide continuous rotation or manual input of the rotation angle, in the proposed design the input is provided by dialing a binary number from 0 to 512 using latching buttons, step 0.7 degrees (in [23] the input was performed with a smaller step, but with less control). Three blocks (Counter) are required to perform continuous rotation and angle control, if it is not required can be replaced by a switch. Through the data buses (Bus A, B) information about the selected angle is entered into the ROM, coefficients from it into the unit of rotation operation. Three two-dimensional rotation devices (Rot. 0-1) and a permanent memory device (ROM) are used, the principle is similar to [23]. At the output there are 4 sampling and storage units (S&H), comparator (Comp.), which allows to realize the display of Poincaré section, upper or lower part of the attractor relative to the section plane (control is conventionally designated as potentiometer Con.2). The set reference signal (r) controls the distance from the selected plane (conventionally labeled as potentiometer Con.1) to the section surface. This mode is enabled by the switch (Sd), change of the projection plane by the switch (S1). A more detailed explanation with many examples and a complete schematic diagram with element ratings and additional information for making the device can be found in [25]. A simpler device, but based on a modern element base, is given in [27]. The appearance of the device is shown in Figure 5 (a), the demonstration of the rotation of the attractor obtained by realizing the Lorentz system by different angles is shown in Figure 5 (b-i) [28].



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Fig. 5. Prototype device pseudo 3D rotation (a), strange atractor at different angles (b-i) [28]

The device is only used to rotate the attractor, in two axes. The rotation is available for 360 degrees. The principle of operation is similar to the works [23, 25]. The device can be divided into the following blocks: signal pre-scaling; rotation angle adjustment; rotation angle expander up to 360 degrees; analog computer to perform mathematical operations; rotation angle indication and generation of control signals. Replacing the ROM with a PIC16F874 microcontroller made it possible to add the indication of the rotation angle for two axes on LED indicators. It also, together with the two digital potentiometers, performs the function of sin/cos potentiometer. Similarly, together with the inverting stages, it provides an extension of the swing angle range up to 2π. The rotation step is 1 degree, and the control is realized on two potentiometers, coarse and fine, for each of the two axes. The frequency range is limited to 20 kHz, but can be extended by replacing the element base. Even greater demonstration can be achieved using stereoscopy methods, which can be used to a greater extent in laboratory work, such methods are described in detail in [29].

Conclusions. The variants of realization of devices for rotating attractors in phase space and displaying the projection on one of the planes were considered. The main element of such devices is a sin/cos potentiometer or its analog. It is used to realize an orthogonal transformation that allows to rotate 3D objects into pseudo-3D. This element can be realized with the help of a bundle: ADC, ROM with coefficient table, digital-to-analog multiplier. Changing the ROM to a microcontroller allows additional functions to be introduced, e.g. indication and rotation angle extension. Thereby reducing the overall size of the device. Combining and modifying the developments of [25, 27] is promising. Transfer of the device to modern element base, increase of frequency range, control of rotation of dynamic chaos generators of greater dimensionality, more control over the rotation angle (input of binary number/controller) and separate functions of the device. For example, Poincaré section, displays of the top and bottom of the attractor (separated by the section plane), distance to the section plane, switching between projections, automatic/manual rotation.

Bibliography:

1. Berkovich, Y., The Van Der Pol Oscillator with a Tunnel Diode, WJERT, 2024, Vol.10, no. 3: P. 40-56.

2. Thio W. J.-C., Sprott J. C. Elegant Circuits: Simple Chaotic Oscillators. World Scientific Publishing Co Pte Ltd, 2022.

3. Мікроелектронний генератор детермінованого хаосу на основі біполярної транзисторної структури з від'ємним диференційним опором / A. Семенов та ін. *Measuring and computing devices in technological processes*. 2023. № 3. С. 206–217. URL: https://doi.org/10.31891/2219-9365-2023-75-24

4. Mathematical modeling of the dynamic chaos mode of electrical oscillations in the colpitts oscillator based on the mosfet / A. O. Semenov et al. *Scientific notes of Taurida National V.I. Vernadsky University. Series: Technical Sciences*. 2022. No. 2. P. 40–47. URL: https://doi.org/10.32838/2663-5941/2022.2/07

5. Simulation of the Chaotic Dynamics of the Deterministic Chaos Transistor Oscillator based on the Hartley Circuit / A. Semenov et al. *2020 IEEE 15th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET)*, Lviv-Slavske, Ukraine, 25–29 February 2020. 2020. URL: https://doi.org/10.1109/tcset49122.2020.235384

6. Petrzela J. Chaos in Analog Electronic Circuits: Comprehensive Review, Solved Problems, Open Topics and Small Example. *Mathematics*. 2022. Vol. 10, no. 21. P. 4108. URL: https://doi.org/10.3390/math10214108

7. Itoh M. Synthesis of electronic circuits for simulating nonlinear dynamics. *International Journal of Bifurcation and Chaos*. 2001. Vol. 11, no. 03. P. 605–653. URL: https://doi.org/10.1142/s0218127401002341

8. Klomkarn K., Sooraksa P. Simple self-instructional modules based on chaotic oscillators: few blocks generating many patterns. *International Journal of Bifurcation and Chaos*. 2011. Vol. 21, no. 05. P. 1469–1491. URL: https://doi.org/10.1142/s021812741102915x

9. Carlà M. The analog computer: Beyond the museum artwork, a tool for studying linear and nonlinear systems. *American Journal of Physics*. 2022. Vol. 90, no. 4. P. 263–272. URL: https://doi.org/10.1119/10.0009634

10. Concise Guide to Chaotic Electronic Circuits / L. Fortuna et al. Springer London, Limited, 2014.

11. Rujzl M., Polak L., Petrzela J. Hybrid Analog Computer for Modeling Nonlinear Dynamical Systems: The Complete Cookbook. *Sensors*. 2023. Vol. 23, no. 7. P. 3599. URL: https://doi.org/10.3390/s23073599

12. Ulmann B. *Analog Computing*. De Gruyter. 2022. 440 p. URL: https://doi.org/10.1515/9783110787740

13. Ulmann B. *Analog and Hybrid Computer Programming*. De Gruyter. 2020. 316 p. URL: https://doi.org/10.1515/9783110787740

14. The analog thing – first steps. URL: https://the-analog-thing.org/THAT_First_Steps.pdf (дата звернення: 23.04.2025)

15. The Analog Paradigm "Model-1.1" User manual. URL: https://analogparadigm.com/downloads/handbook.pdf (дата звернення: 23.04.2025)

16. Two-Dimensional Rotation of Chaotic Attractors: Demonstrative Examples and FPGA Realization / W. S. Sayed et al. *Circuits, Systems, and Signal Processing*. 2019. Vol. 38, no. 10. P. 4890–4903. URL: https://doi.org/10.1007/s00034-019-01096-z

17. CORDIC-Based FPGA Realization of a Spatially Rotating Translational Fractional-Order Multi-Scroll Grid Chaotic System / W. S. Sayed et al. *Fractal and Fractional*. 2022. Vol. 6, no. 8. P. 432. URL: https://doi.org/10.3390/fractalfract6080432

18. A Deterministic Chaos Ring Oscillator Based on a MOS Transistor Structure with Negative Differential Resistance / A. Semenov et al. *2019 IEEE International Scientific-Practical Conference Problems of Infocommunications, Science and Technology (PIC S&T)*, Kyiv, Ukraine, 8–11 October 2019. 2019. URL: https://doi.org/10.1109/picst47496.2019.9061330

19. Numerical study of the deterministic chaos oscillator with a differential integral element on the colpitts circuit / A. Semenov et al. *2018 14th International Conference on Advanced Trends in Radioelecrtronics, Telecommunications and Computer Engineering (TCSET)*, Lviv-Slavske, Ukraine, 20–24 February 2018. 2018. URL: https://doi.org/10.1109/tcset.2018.8336329

20. MacKay D. M. Analogue computing at ultra-high speed: An experimental and theoretical study. New York : Wiley, 1962. 395 p.

21. Shapley R., Rossetto M. An electronic visual stimulator. *Behavior Research Methods & Instrumentation*. 1976. Vol. 8, no. 1. P. 15–20. URL: https://doi.org/10.3758/bf03201656

22. Heeley D. W. A rotator for X/Y oscilloscope displays. *Behavior Research Methods & Instrumentation*. 1983. Vol. 15, no. 5. P. 503–507. URL: https://doi.org/10.3758/bf03203696

23. Higgs A. A digital image rotating system. *Journal of Physics E: Scientific Instruments*. 1982. Vol. 15, no. 3. P. 266–267. URL: https://doi.org/10.1088/0022-3735/15/3/001

24. Kennedy M. P. Hardware toolkit for studying chaos: a live demonstration of nonlinear dynamics instrumentation with audience participation. *34th Midwest Symposium on Circuits and Systems*, Monterey, CA, USA. URL: https://doi.org/10.1109/mwscas.1991.252002

25. Chua L. O., Sugawara T. Panoramic views of strange attractors. *Proceedings of the IEEE*. 1987. Vol. 75, no. 8. P. 1107–1120. URL: https://doi.org/10.1109/proc.1987.13853

26. Matsumoto T. A chaotic attractor from Chua's circuit. *IEEE Transactions on Circuits and Systems*. 1984. Vol. 31, no. 12. P. 1055–1058. URL: https://doi.org/10.1109/tcs.1984.1085459

27. A 3-Dimensional Projective Unit URL: https://glensstuff.com/3dpu/3dpu.htm (дата звернення: 24.04.2025)

28. A Lorenz Attractor Circuit URL: https://glensstuff.com/lorenzattractor/lorenz.htm (дата звернення: 24.04.2025)

29. Tilton H. B. The 3-D oscilloscope: A practical manual and guide. Englewood Cliffs, N.J : Prentice-Hall, 1987. 231 p.

References:

1. Berkovich, Y. (2024). The Van Der Pol Oscillator with a Tunnel Diode. *World Journal of Engineering Research and Technology*, 10(3), 40-56.

2. Thio, W. J.-C., & Sprott, J. C. (2022). *Elegant Circuits: Simple Chaotic Oscillators*. World Scientific Publishing Co Pte Ltd.

3. Semenov, A., Pinaiev, B., Khloba, A., Shurkhal, M., & Olkhovych, V. (2023). Mikroelektronnyi henerator determinovanoho khaosu na osnovi bipoliarnoi tranzystornoi struktury z vidiemnym dyferentsiinym oporom. Measuring and Computing Devices in Technological Processes, (3), 206–217. https://doi.org/10.31891/2219-9365-2023-75-24

4. Semenov, A. O., Semenova, O. O., Pinaiev, B. O., Kulias, R. O., & Shpylovyi, O. O. (2022). Mathematical modeling of the dynamic chaos mode of electrical oscillations in the colpitts oscillator based on the mosfet. *Scientific Notes of Taurida National V.I. Vernadsky University. Series: Technical Sciences*, (2), 40–47. https://doi.org/10.32838/2663-5941/2022.2/07

5. Semenov, A., Havrilov, D., Volovik, A., Baraban, S., Savytskyi, A., & Zviahin, O. (2020). Simulation of the chaotic dynamics of the deterministic chaos transistor oscillator based on the hartley circuit. У *2020 IEEE 15th international conference on advanced trends in radioelectronics, telecommunications and computer engineering (TCSET)*. IEEE. https://doi.org/10.1109/tcset49122.2020.235384

6. Petrzela, J. (2022). Chaos in analog electronic circuits: Comprehensive review, solved problems, open topics and small example. *Mathematics*, *10*(21), 4108. https://doi.org/10.3390/math10214108

7. Itoh, M. (2001). Synthesis of electronic circuits for simulating nonlinear dynamics. *International Journal of Bifurcation and Chaos*, *11*(03), 605–653. https://doi.org/10.1142/s0218127401002341

8. Klomkarn, K., & Sooraksa, P. (2011). Simple self-instructional modules based on chaotic oscillators: Few blocks generating many patterns. *International Journal of Bifurcation and Chaos*, *21*(05), 1469–1491. https://doi.org/10.1142/s021812741102915x

9. Carlà, M. (2022). The analog computer: Beyond the museum artwork, a tool for studying linear and nonlinear systems. *American Journal of Physics*, *90*(4), 263–272. https://doi.org/10.1119/10.0009634

10. Fortuna, L., Frasca, M., Buscarino, A., & Sciuto, G. (2014). *Concise guide to chaotic electronic circuits*. Springer London, Limited.

11. Rujzl, M., Polak, L., & Petrzela, J. (2023). Hybrid analog computer for modeling nonlinear dynamical systems: The complete cookbook. *Sensors*, *23*(7), 3599. https://doi.org/10.3390/s23073599

12. Ulmann B. (2022). У *Analog computing*. De Gruyter. https://doi.org/10.1515/9783110787740
13. Ulmann B. (2020). У *Analog and hybrid computer programming*. De Gruyter Oldenbourg. https://doi.org/10.1515/9783110787740
14. The analog thing – first steps. (2023). Available at: https://the-analog-thing.org/THAT_First_Steps.pdf (date of access: 23.04.2025)
15. The Analog Paradigm "Model-1.1" User manual. (2019). Available at: https://analogparadigm.com/downloads/handbook.pdf (date of access: 23.04.2025)
16. Sayed, W. S., Radwan, A. G., Elnawawy, M., Orabi, H., Sagahyroon, A., Aloul, F., Elwakil, A. S., Fahmy, H. A., & El-Sedeek, A. (2019). Two-Dimensional rotation of chaotic attractors: Demonstrative examples and FPGA realization. *Circuits, Systems, and Signal Processing*, *38*(10), 4890–4903. https://doi.org/10.1007/s00034-019-01096-z
17. Sayed, W. S., Roshdy, M., Said, L., Herencsar, N., & Radwan, A. (2022). CORDIC-Based FPGA realization of a spatially rotating translational fractional-order multi-scroll grid chaotic system. *Fractal and Fractional*, *6*(8), 432. https://doi.org/10.3390/fractalfract6080432
18. Semenov, A., Osadchuk, O., Semenova, O., Koval, K., Baraban, S., & Savytskyi, A. (2019). A deterministic chaos ring oscillator based on a MOS transistor structure with negative differential resistance. У *2019 IEEE international scientific-practical conference problems of infocommunications, science and technology (PIC S&T)*. IEEE. https://doi.org/10.1109/picst47496.2019.9061330
19. Semenov, A., Koval, K., Savytskyi, A., Zviahin, O., & Baraban, S. (2018). Numerical study of the deterministic chaos oscillator with a differential integral element on the colpitts circuit. У *2018 14th international conference on advanced trends in radioelecrtronics, telecommunications and computer engineering (TCSET)*. IEEE. https://doi.org/10.1109/tcset.2018.8336329
20. MacKay, D. M. (1962). *Analogue computing at ultra-high speed: An experimental and theoretical study*. Wiley.
21. Shapley, R., & Rossetto, M. (1976). An electronic visual stimulator. *Behavior Research Methods & Instrumentation*, *8*(1), 15–20. https://doi.org/10.3758/bf03201656
22. Heeley, D. W. (1983). A rotator for X/Y oscilloscope displays. *Behavior Research Methods & Instrumentation*, *15*(5), 503–507. https://doi.org/10.3758/bf03203696
23. Higgs, A. (1982). A digital image rotating system. *Journal of Physics E: Scientific Instruments*, *15*(3), 266–267. https://doi.org/10.1088/0022-3735/15/3/001
24. Kennedy, M. P. (б. д.). Hardware toolkit for studying chaos: A live demonstration of nonlinear dynamics instrumentation with audience participation. У *34th midwest symposium on circuits and systems*. IEEE. https://doi.org/10.1109/mwscas.1991.252002
25. Chua, L. O., & Sugawara, T. (1987). Panoramic views of strange attractors. *Proceedings of the IEEE*, *75*(8), 1107–1120. https://doi.org/10.1109/proc.1987.13853
26. Matsumoto, T. (1984). A chaotic attractor from Chua's circuit. *IEEE Transactions on Circuits and Systems*, *31*(12), 1055–1058. https://doi.org/10.1109/tcs.1984.1085459
27. A 3-Dimensional Projective Unit. (2013). Available at: https://glensstuff.com/3dpu/3dpu.htm (date of access: 24.04.2025)
28. A Lorenz Attractor Circuit. (2013). Available at: https://glensstuff.com/lorenzattractor/lorenz.htm (date of access: 24.04.2025)
29. Tilton, H. B. (1987). *The 3-D oscilloscope: A practical manual and guide*. Prentice-Hall.